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1997 J. Phys.: Condens. Matter 9 693

(http://iopscience.iop.org/0953-8984/9/3/009)

J. Phys.: Condens. Matter **9** (1997) 693–701. Printed in the UK PII: S0953-8984(97)77271-0

An effective-Hamiltonian approach to the study of the interference effect in macroscopic magnetic coherence

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Received 20 August 1996, in final form 10 October 1996

Abstract. The quantum interference in a magnetic anisotropy model with four easy directions is studied within the spin coherent-state path-integral formalism. When the particle possesses half-integer spin, the Euclidean propagators reduce to that of a two-level system, indicating degeneracy of the states. The results are shown to be easily reproduced by introducing an effective tunnelling Hamiltonian, where the topological phase factor is properly incorporated. This effective-Hamiltonian approach is demonstrated to be equivalent to the dilute-instanton approximation. The discussions are then generalized to the N-well cases, and the symmetry of the effective Hamiltonian is analysed. Degeneracies of the tunnelling levels are discussed in detail. The quenching result obtained previously in the case of a double well can be recovered by a simple diagonalization of the effective Hamiltonian.

1. Introduction

The study of macroscopic magnetic tunnelling has been a subject of extensive research in recent years [1]. It was proposed [2–4] that the magnetization of small particles can tunnel between degenerate easy directions through the anisotropy barrier. Experiments involving resonance measurements [5], magnetic relaxation [6] and hysteresis loop study [7] for various systems suggest that the tunnelling is observable.

For magnetic anisotropy with two or more degenerate easy directions where quantum coherence occurs, there are usually different tunnelling paths connecting the same initial and final configurations. These paths will interfere with each other, and due to the topological phase factor, as shown by Loss, DiVincenzo and Grinstein [8] and von Delft and Henley [9], can lead to quenching (complete cancellation) of tunnelling if the particle possesses half-integer spin in the case of a double well. This suppression of the tunnelling splitting turns out to be in agreement with the Kramers theorem, which demands that a state with its time-reversed counterpart should be degenerate if the Hamiltonian is time-reversal invariant. In fact, the Kramers degeneracy in the spin-tunnelling problem was first mentioned by Enz and Schilling [2] and van Hemmen and Sütö [3], while the work of Loss, DiVincenzo and Grinstein [8] and von Delft and Henley [9] demonstrated that the degeneracy can be understood as a quantum interference effect. Later the quantum interference effects are studied in the presence of an external magnetic field, with the field along either the hard [10], easy [11] or medium axis [12].

In these studies, the Euclidean transition amplitude is written as a spin coherent-state path integral, and the saddle-point approximation is then applied. In this paper we further study a four-well (i.e., four consecutive minima within one period) problem using this method. When the particle possesses half-integer spin, the Euclidean propagators are found to reduce to that of a two-level system, indicating degeneracy of the states. We show how the results obtained in the path-integral formalism can be recovered by introducing an effective tunnelling Hamiltonian. This approach is a development of the tunnelling Hamiltonian of Leggett *et al* [13], where the topological phase factor is properly incorporated. The effective-Hamiltonian approach is demonstrated to be equivalent to the dilute-instanton approximation of Coleman [14], while this approach has the advantage of being very simple and direct. It permits us to discuss the degeneracies of the levels in detail. The generalization to the N-well situations is presented, where we get the low-lying tunnelling spectrum conveniently—which is usually not easy to obtain within the path-integral formalism. The symmetry of the effective Hamiltonian is analysed, and the degeneracy of the low-lying energy levels which is a result of quantum interference is shown to follow from a 'time-reversal' symmetry of the effective Hamiltonian. The quenching result in the case of a double well [8, 9] can be reproduced by a simple diagonalization of the effective Hamiltonian.

2. The four-well problem

We first consider a problem with four consecutive minima in a period. Physically this can be realized in a particle of tetragonal anisotropy symmetry with appropriate anisotropic constants so that the basal plane is the easy plane. The anisotropy energy can be written as [15]

$$E(\theta,\phi) = -K_1 \sin^2\theta + [K_2 - K_2' \cos(4\phi)] \sin^4\theta \tag{1}$$

where K_1, K_2, K_2' are anisotropic constants satisfying $K_1, K_2' > 0$ and $K_1 \gg K_2, K_2'$. The ground state of the particle corresponds to $\boldsymbol{M}$ pointing in one of the four easy directions: $\theta = \pi/2$, $\phi = 0, \pi/2, \pi$, and $3\pi/2$, or we can say that the configuration space is a circle, and the four configurations have lowest energy. If we denote the four states as $|1\rangle, |2\rangle, |3\rangle$, and $|4\rangle$, other minima repeat the four states with period 2π. In this paper we restrict our attention to the ground-state tunnelling since it is expected to be dominant at low temperatures.

2.1. *Calculation of the Euclidean transition amplitude*

As usual, the Euclidean transition amplitude (or propagator) is written as a spin coherent-state path integral [16]:

$$\langle j'|\mathrm{e}^{-(H/\hbar)T}|j\rangle = \int \mathrm{D}\Omega\ \mathrm{e}^{-S_E/\hbar} \tag{2}$$

where $|j\rangle, |j'\rangle$ denote any two of the four states, $S_E = \int_{-T/2}^{T/2} L\ \mathrm{d}\tau$ is the Euclidean action, and L is the Lagrangian:

$$L = \mathrm{i}S\dot{\phi}(1 - \cos\theta) + E. \tag{3}$$

Here S is the spin quantum number. The first term of equation (3) arises from the nonorthogonality of coherent states. It has a topological origin and is crucial for the study of quantum interference. In fact it is this term that makes integer and half-integer spin quantum numbers different, as first discussed by Haldane [17]. It should also be mentioned that one must be very careful when using the spin coherent-state path integral, since in this approach the discontinuous paths are ignored which is unjustified [18]. Sometimes incorrect results are yielded, as demonstrated by Enz and Schilling [2].

Using the dilute-instanton approximation [14], the propagator can be expressed as

$$\langle j'|\mathrm{e}^{-(H/\hbar)T}|j\rangle = \sqrt{\frac{\omega}{\pi\hbar}}\mathrm{e}^{-\omega T/2}\sum_{m,n}^{m-n=j'-j(\mathrm{mod}\ 4)}\frac{(JKT\mathrm{e}^{-S_0/\hbar}\mathrm{e}^{-\mathrm{i}S\pi/2})^m(JKT\mathrm{e}^{-S_0/\hbar}\mathrm{e}^{\mathrm{i}S\pi/2})^n}{m!n!}. \tag{4}$$

Here ω is the zero-point frequency in the well, S_0 the instanton action, J a Jacobian, and for the definition of K the reader is referred to Coleman [14]. The instanton solution mapping $|1\rangle$ to $|2\rangle$ is found to be

$$\phi(\tau) = \frac{1}{2}\arccos\left[-\sqrt{\frac{K_1 - 2K_2 - 2K_2'}{(K_1 - 2K_2 + 2K_2') - 4K_2'\tanh^2\omega\tau}}\tanh\omega\tau\right]. \tag{5}$$

Then one gets

$$S_0/\hbar = S\int_0^{\pi/2}\sqrt{\frac{K_2'(1-\cos 4\phi)}{K_1 - 2K_2 + 2K_2'\cos 4\phi}}\ \mathrm{d}\phi = S\sqrt{\frac{2K_2'}{K_1 + 2K_2'}}$$

and

$$\hbar\Delta \equiv JK\mathrm{e}^{-S_0/\hbar} = 4\sqrt{\frac{K_2'}{S\pi}}\sqrt{2K_1K_2'}\mathrm{e}^{-S\sqrt{2K_2'/(K_1+2K_2)}}. \tag{6}$$

The propagators from $|1\rangle$ to the other states are as follows:

$$\begin{aligned}
\langle 1|\mathrm{e}^{-(H/\hbar)T}|1\rangle &= \frac{1}{2}\sqrt{\frac{\omega}{\pi\hbar}}\mathrm{e}^{-\omega T/2}\left[\cosh\left(2\hbar\Delta T\cos\frac{S\pi}{2}\right) + \cosh\left(2\hbar\Delta T\sin\frac{S\pi}{2}\right)\right]\\
\langle 2|\mathrm{e}^{-(H/\hbar)T}|1\rangle &= \frac{1}{2}\sqrt{\frac{\omega}{\pi\hbar}}\mathrm{e}^{-\omega T/2}\left[\sinh\left(2\hbar\Delta T\cos\frac{S\pi}{2}\right) - \mathrm{i}\sinh\left(2\hbar\Delta T\sin\frac{S\pi}{2}\right)\right]\\
\langle 3|\mathrm{e}^{-(H/\hbar)T}|1\rangle &= \frac{1}{2}\sqrt{\frac{\omega}{\pi\hbar}}\mathrm{e}^{-\omega T/2}\left[\cosh\left(2\hbar\Delta T\cos\frac{S\pi}{2}\right) - \cosh\left(2\hbar\Delta T\sin\frac{S\pi}{2}\right)\right]\\
\langle 4|\mathrm{e}^{-(H/\hbar)T}|1\rangle &= \frac{1}{2}\sqrt{\frac{\omega}{\pi\hbar}}\mathrm{e}^{-\omega T/2}\left[\sinh\left(2\hbar\Delta T\cos\frac{S\pi}{2}\right) + \mathrm{i}\sinh\left(2\hbar\Delta T\sin\frac{S\pi}{2}\right)\right].
\end{aligned} \tag{7}$$

One can see that when S is a half-integer, the propagators behave like that of a two-level system, indicating degeneracy of the states. Moreover, the propagator $\langle 3|\mathrm{e}^{-(H/\hbar)T}|1\rangle$ vanishes if S is a half-integer, in accordance with the Kramers theorem. We will show how to understand these quantum interference effects through an effective Hamiltonian in the following.

2.2. The effective Hamiltonian

Now we show how the results can be obtained using an alternative method. We introduce an effective Hamiltonian

$$H_{eff} = -\hbar\Delta M \tag{8}$$

where M is a linear operator defined by

$$M|j\rangle = p|j+1\rangle + q|j-1\rangle. \tag{9}$$

The matrix form of M is

$$[M] = (\langle i|M|j\rangle) = \begin{bmatrix} 0 & q & 0 & p \\ p & 0 & q & 0 \\ 0 & p & 0 & q \\ q & 0 & p & 0 \end{bmatrix}. \tag{10}$$

H_{eff} is Hermitian if $p = q^*$ and can be diagonalized. In fact they should be specified by

$$p = q^* = \mathrm{e}^{-\mathrm{i}S\pi/2}. \tag{11}$$

Then we obtain the eigenstates

$$\begin{aligned} |0) &= \frac{1}{2}(|1\rangle + |2\rangle + |3\rangle + |4\rangle) \\ |1) &= \frac{1}{2}(|1\rangle + \mathrm{i}|2\rangle - |3\rangle - \mathrm{i}|4\rangle) \\ |2) &= \frac{1}{2}(|1\rangle - |2\rangle + |3\rangle - |4\rangle) \\ |3) &= \frac{1}{2}(|1\rangle - \mathrm{i}|2\rangle - |3\rangle + \mathrm{i}|4\rangle) \end{aligned} \tag{12}$$

with the corresponding eigenvalues

$$E = -2\overline{h}\,\Delta\cos\left(\frac{S\pi}{2}\right), -2\overline{h}\,\Delta\sin\left(\frac{S\pi}{2}\right), 2\overline{h}\,\Delta\cos\left(\frac{S\pi}{2}\right), 2\overline{h}\,\Delta\sin\left(\frac{S\pi}{2}\right). \tag{13}$$

One can check that we obtain the same result as equation (6) by making use of $I = |0)(0| + |1)(1| + |2)(2| + |3)(3|$. On the other hand, when S is a half-integer, there are only two different levels with double degeneracy. This is a result of the interference.

2.3. The equivalence

Equation (8) can be viewed as indicating that in one step $|j\rangle$ goes to $|j+1\rangle$ forward with weight p, and to $|j-1\rangle$ backward with weight q. We get after N steps

$$\begin{aligned} M^N|1\rangle = &\sum_{m,n}^{m+n=N, m-n=0 (\mathrm{mod}\ 4)} \binom{N}{m} p^m q^n |1\rangle + \sum_{m,n}^{m+n=N, m-n=1 (\mathrm{mod}\ 4)} \binom{N}{m} p^m q^n |2\rangle \\ &+ \sum_{m,n}^{m+n=N, m-n=2 (\mathrm{mod}\ 4)} \binom{N}{m} p^m q^n |3\rangle + \sum_{m,n}^{m+n=N, m-n=3 (\mathrm{mod}\ 4)} \binom{N}{m} p^m q^n |4\rangle. \end{aligned}$$

Thus

$$\begin{aligned} \frac{M^N}{N!}|1\rangle = &\sum_{m,n}^{m+n=N, m-n=0 (\mathrm{mod}\ 4)} \frac{p^m q^n}{m!n!}|1\rangle + \sum_{m,n}^{m+n=N, m-n=1 (\mathrm{mod}\ 4)} \frac{p^m q^n}{m!n!}|2\rangle \\ &+ \sum_{m,n}^{m+n=N, m-n=2 (\mathrm{mod}\ 4)} \frac{p^m q^n}{m!n!}|3\rangle + \sum_{m,n}^{m+n=N, m-n=3 (\mathrm{mod}\ 4)} \frac{p^m q^n}{m!n!}|4\rangle. \end{aligned}$$

Summing over N from 0 to ∞, we have

$$\mathrm{e}^{-(H/\overline{h})T}|1\rangle = \sum_{m,n}^{m-n=0 (\mathrm{mod}\ 4)} \frac{(\overline{h}\Delta T p)^m (\overline{h}\Delta T q)^n}{m!n!}|1\rangle + \sum_{m,n}^{m-n=1 (\mathrm{mod}\ 4)} \frac{(\overline{h}\Delta T p)^m (\overline{h}\Delta T q)^n}{m!n!}|2\rangle$$

$$+\sum_{m,n}^{m-n=2(\text{mod }4)}\frac{(\bar{h}\Delta Tp)^m(\bar{h}\Delta Tq)^n}{m!n!}|3\rangle+\sum_{m,n}^{m-n=3(\text{mod }4)}\frac{(\bar{h}\Delta Tp)^m(\bar{h}\Delta Tq)^n}{m!n!}|4\rangle. \tag{14}$$

One then obtains the same result as equation (6). The absence of the prefactor here is because we have set the diagonal elements of the effective Hamiltonian to zero.

There is yet another way of computing the propagator. Since

$$M|0)=(p+q)|0)$$
$$M|1)=-\mathrm{i}(p-q)|1)$$
$$M|2)=-(p+q)|2)$$
$$M|3)=\mathrm{i}(p-q)|3)$$

one has

$$\mathrm{e}^M|1\rangle=\frac{1}{2}\mathrm{e}^M\{|0)+|1)+|2)+|3)\}=\frac{1}{2}\{\mathrm{e}^{p+q}|0)+\mathrm{e}^{-\mathrm{i}(p-q)}|1)+\mathrm{e}^{-(p+q)}|2)+\mathrm{e}^{\mathrm{i}(p-q)}|3)\}$$
$$=\frac{1}{2}\{[\cosh(p+q)+\cos(p-q)]|1\rangle+[\sinh(p+q)+\sin(p-q)]|2\rangle$$
$$+[\cosh(p+q)-\cos(p-q)]|3\rangle+[\sinh(p+q)+\sin(p-q)]|4\rangle\}.$$

Then the same results are obtained.

The effective Hamiltonian has at least two symmetry operations under which it is invariant: translation and 'time reversal'. Using these one immediately obtains the eigenvalues and eigenstates, and the degeneracies of the eigenstates can be analysed in detail. These will be presented in the discussion of N-well cases.

3. Generalization to N-well situations

There are physical systems where there are several wells within one period. For instance, a system with hexagonal symmetry can possibly possess six easy axes in the basal plane [15]. In this section, we generalize the previous discussions to situations with N wells.

Introducing the effective Hamiltonian

$$H_{eff}=-\bar{h}\Delta M \tag{15}$$

where $\bar{h}\Delta$ is the tunnelling matrix element and

$$M|j\rangle=p|j+1\rangle+q|j-1\rangle \qquad j=1,2,\ldots,N \tag{16}$$

with cyclic boundary conditions $|0\rangle=|N\rangle$, $|n+1\rangle=|1\rangle$. Here

$$q^*=p=\mathrm{e}^{-\mathrm{i}2S\pi/N}. \tag{17}$$

Define the translation or cyclic operator T as

$$T|j\rangle=|j+1\rangle \qquad j=1,2,\ldots,N \tag{18}$$

with $|j\rangle=|j(\text{mod }N)\rangle$. It is easy to see that T commutes with H_{eff}, so they have the same eigenstates which are characterized by the irreducible representation of G. Therefore $T|l)=\mathrm{e}^{\mathrm{i}k_l}|l)$ and

$$H_{eff}|l)=-\bar{h}\Delta(pT+qT^{-1})|l)=-\bar{h}\Delta(p\mathrm{e}^{\mathrm{i}k_l}+q\mathrm{e}^{-\mathrm{i}k_l})|l)=-2\bar{h}\Delta\cos\frac{2\pi}{N}(S-l)|l). \tag{19}$$

On the other hand

$$\langle j|l)=\langle j-1|T^{-1}|l)=\mathrm{e}^{-\mathrm{i}k_l}\langle j-1|l)=\mathrm{e}^{-\mathrm{i}2k_l}\langle j-2|l)=\cdots=\mathrm{e}^{-\mathrm{i}k_l(j-1)}\langle 1|l). \tag{20}$$

Taking $\langle 1|l) = 1/\sqrt{N}$ as the proper normalization constant, one has

$$|l) = \sum_{j=1}^{N} |j\rangle\langle j|l) = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} \mathrm{e}^{-\mathrm{i}(2\pi/N)(j-1)l} |j\rangle \qquad l = 0, 1, 2, \ldots, N-1. \tag{21}$$

or

$$|j\rangle = \sum_{l=0}^{N-1} |l)(l|j\rangle = \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} \mathrm{e}^{\mathrm{i}(2\pi/N)(j-1)l} |j\rangle \qquad j = 1, 2, \ldots, N. \tag{22}$$

Now we can calculate the propagators

$$\begin{aligned} \langle j'|\mathrm{e}^{-(H/\hbar)T}|j\rangle &= \sum_{l=0}^{N-1} \langle j'|l)(l\mathrm{e}^{-(H/\hbar)T}|j\rangle \mathrm{e}^{-E_l T/\hbar} \\ &= \frac{1}{N} \sum_{l=0}^{N-1} \sum_{l=0}^{N-1} \mathrm{e}^{\mathrm{i}(2\pi/N)(j-j')l} \mathrm{e}^{2\Delta T \cos(2\pi/N)(S-l)}. \end{aligned} \tag{23}$$

One can observe translation invariance of the propagator $\langle j' + n|\mathrm{e}^{-(H/\hbar)T}|j + n\rangle = \langle j'|\mathrm{e}^{-(H/\hbar)T}|j\rangle$ (n is an integer) since $THT^{-1} = H$.

To explore possible ‘time-reversal’ invariance [19] we introduce the operator $\tau = KC$ and demand that $\tau H \tau^{-1} = H$, where C is the conjugation operator which is anti-linear, and K is unitary. If we assume that K is diagonal, then it can be chosen as

$$K|j\rangle = \mathrm{e}^{\mathrm{i}(4S\pi/N)(N-j+1)}|j\rangle. \tag{24}$$

A smarter choice is also allowable:

$$K = \exp(\mathrm{i}S\pi J_z)$$

where

$$J_z = \begin{bmatrix} J & & & & \\ & J-1 & & & \\ & & \cdots & & \\ & & & -J+1 & \\ & & & & -J \end{bmatrix}$$

with J defined as $(N-1)/2$.

To examine the possible selection rule of $\langle j'|\mathrm{e}^{-(H/\hbar)T}|j\rangle$ resulting from the ‘time-reversal’ symmetry, we use

$$\begin{aligned} \langle j'|\mathrm{e}^{-(H/\hbar)T}|j\rangle^* &= \langle j'|\tau^*\tau\mathrm{e}^{-(H/\hbar)T}|j\rangle = \langle j'|\tau^*\mathrm{e}^{-(H/\hbar)T}\tau|j\rangle = \langle j'|K^\dagger\mathrm{e}^{-(H/\hbar)T}K|j\rangle \\ &= \mathrm{e}^{\mathrm{i}(4S\pi/N)(j'-j)}\langle j'|\mathrm{e}^{-(H/\hbar)T}|j\rangle. \end{aligned}$$

If $\langle j'|\mathrm{e}^{-(H/\hbar)T}|j\rangle$ is real and $\mathrm{e}^{\mathrm{i}(4S\pi/N)(j'-j)} \neq 1$, then $\langle j'|\mathrm{e}^{-(H/\hbar)T}|j\rangle$ vanishes. This can be realized when N *is even and* $j' - j = N/2$, $S =$ *half-integer*. Or, when N is even and S is a half-integer, we have

$$\begin{aligned} \langle N/2+1|\mathrm{e}^{-(H/\hbar)T}|1\rangle &= \langle N/2+2|\mathrm{e}^{-(H/\hbar)T}|2\rangle = \cdots = \langle N|\mathrm{e}^{-(H/\hbar)T}|N/2\rangle \\ &= \langle 1|\mathrm{e}^{-(H/\hbar)T}|N/2+1\rangle = \langle 2|\mathrm{e}^{-(H/\hbar)T}|N/2+2\rangle = \cdots = \langle N/2|\mathrm{e}^{-(H/\hbar)T}|N\rangle \\ &= 0. \end{aligned} \tag{25}$$

We now come to the degeneracy of the tunnelling levels from the ‘time-reversal’ symmetry. Since $\tau H \tau^{-1} = H$, $\tau|l)$ and $|l)$ belongs to the same level, and we can show that $\tau|l) = |2S - l)$:

$$
\begin{aligned}
\tau|l) &= KC\frac{1}{\sqrt{N}}\sum_{j=1}^{N} e^{-i(2\pi/N)(j-1)l}|j\rangle = K\frac{1}{\sqrt{N}}\sum_{j=1}^{N} e^{i(2\pi/N)(j-1)l}|j\rangle \\
&= \frac{1}{\sqrt{N}}\sum_{j=1}^{N} e^{i(2\pi/N)(j-1)l+i(4S\pi/N)(N-j+1)}|j\rangle \\
&= \frac{1}{N}\sum_{j=1}^{N}\sum_{l'=0}^{N-1} e^{i(2\pi/N)(j-1)l+i(4S\pi/N)(N-j+1)+i(2\pi/N)(j-1)l'}|l') \\
&= \frac{1}{N}\sum_{j=1}^{N}\sum_{l'=0}^{N-1} e^{i(2\pi/N)j(l+l'-2S)+i(2\pi/N)(-l'-l+2S+2NS)}|l') \\
&= \sum_{l'=0}^{N-1} e^{i(2\pi/N)(-l'-l+2S+2NS)}\delta_{l+l',2S}|l') \\
&= e^{i4S\pi}|2S-l) = |2S-l).
\end{aligned}
\tag{26}
$$

Therefore the conclusion is that $|l)$ *and* $|2S - l)$ *are degenerate, if* $|l) \neq |2S - l)$. Use can be made of this to study the degeneracy of the tunnelling spectrum for different spin quantum numbers S and different number of wells N. First we note that

$$|l \pm N) = |l) \tag{27}$$

and

$$\left|2\left(S + \frac{N}{2}\right) - l\right) = |2S - l + N) = |2S - l) \tag{28}$$

which means that it is sufficient to study $S = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots, N/2 - 1$. There are altogether four cases.

3.1. N = *even*, S = *half-integer*

All eigenstates are paired with double degeneracy by

$$
\begin{aligned}
&|1), |2S-1) \\
&|2), |2S-2) \\
&\vdots \\
&|2S), |N(0)) \\
&|2S+1), |N-1) \\
&\vdots \\
&\left|\frac{N}{2} + S - \frac{1}{2}\right), \left|\frac{N}{2} + S + \frac{1}{2}\right).
\end{aligned}
$$

3.2. N = *even*, S = *integer*

$|S)$ and $|S + N/2)$ are non-degenerate; all other $N - 2$ states are paired with double degeneracy:

$$|S+1), |S-1)$$

$$|S+2),\ |S-2)$$
$$\vdots$$
$$|2S),\ |N(0))$$
$$|2S+1),\ |N-1)$$
$$\vdots$$
$$\left|\frac{N}{2}+S-1\right),\ \left|\frac{N}{2}+S+1\right).$$

3.3. $N = odd$, $S = half\text{-}integer$

$|S+N/2)$ is non-degenerate; the other $N-1$ states are doubly degenerate:

$$\left|S+\frac{N}{2}+1\right),\ \left|S+\frac{N}{2}-1\right)$$
$$\left|S+\frac{N}{2}+2\right),\ \left|S+\frac{N}{2}-2\right)$$
$$\vdots$$
$$|N),\ |N+2S)$$
$$|1),\ |N+2S+1)$$
$$\vdots$$
$$\left|\frac{N}{2}+S-1\right),\ \left|\frac{N}{2}+S+1\right).$$

3.4. $N = odd$, $S = integer$

$|S)$ is non-degenerate; all other $N-1$ states are paired in a doublet:

$$|S+1),\ |S-1)$$
$$|S+2),\ |S-2)$$
$$\vdots$$
$$|2S),\ |0)$$
$$|2S+1),\ |N-1)$$
$$\vdots$$
$$\left|\frac{N}{2}+S-1\right),\ \left|\frac{N}{2}+S+1\right).$$

4. Summary

In this paper we have studied the interference effect in macroscopic magnetic coherence in the case of four consecutive minima within one period. We see that when S is a half-integer, the propagators behave like that of a two-level system. It is shown that the results obtained in the path-integral formalism can be recovered by introducing an effective tunnelling Hamiltonian, if the topological phase factor is properly incorporated. This

Hamiltonian approach is demonstrated to be equivalent to the dilute-instanton approximation. This permits us to discuss the tunnelling spectrum conveniently. We find two symmetry operations under which the effective Hamiltonian is invariant. The discussions are generalized to N-well cases, where the eigenstates and eigenvalues are obtained through group theoretical analysis. The degeneracies of the tunnelling spectrum are discussed in detail.

We end this paper with an application of the effective Hamiltonian. In the case of a double well, H_{eff} is

$$H_{eff} = -\hbar\Delta \begin{bmatrix} 0 & p+q \\ p+q & 0 \end{bmatrix}$$

where $q^* = p = \mathrm{e}^{-\mathrm{i}S\pi}$. One observes that the two eigenvalues are

$$E = \pm\hbar\Delta(p+q) = \pm 2\hbar\Delta\cos(S\pi).$$

When S is a half-integer, they are degenerate. This reproduces the quenching result for half-integer spin [8, 9], and it is indeed a simple derivation. Our previous calculation [12] can also be simplified in the same way.

Acknowledgments

Stimulating discussions with Su-Peng Kou, Yun-Bo Zhang and Professor J Q Liang are acknowledged. This work was supported by the National Natural Science Foundation of China.

References

[1] For a description of recent progress, see
Gunther L and Barbara B (ed) 1995 *Quantum Tunneling of Magnetization (Proc. NATO Conf.)* (Dordrecht: Kluwer)
[2] Enz M and Schilling R 1986 *J. Phys. C: Solid State Phys.* **19** 1765, L711
[3] van Hemmen J L and Sütö A 1986 *Europhys. Lett.* **1** 481; 1986 *Physica* B **141** 37
[4] Chudnovsky E M and Gunther L 1988 *Phys. Rev. Lett.* **60** 661
[5] Awschalom D D, DiVincenzo D P and Smyth J F 1992 *Science* **258** 414 and references therein
[6] Barbara B, Sampaio L C, Wegrowe J E, Ratnam B A, Marchand A, Paulsen C, Novak M A, Tholence J L, Uehara M and Fruchart D 1993 *J. Appl. Phys.* **73** 6703 and references therein
Tejada J and Zhang X X 1993 *J. Appl. Phys.* **73** 6709 and references therein
[7] Friedman J R, Sarachik M P, Tejada J and Ziolo R 1996 *Phys. Rev. Lett.* **76** 3830
[8] Loss D, DiVincenzo D P and Grinstein G 1992 *Phys. Rev. Lett.* **69** 3232
[9] von Delft J and Henley C L 1992 *Phys. Rev. Lett.* **69** 3236
[10] Garg A 1993 *Europhys. Lett.* **22** 205
[11] Chudnovsky E M and DiVincenzo D P 1993 *Phys. Rev.* B **48** 10 548
[12] Wang X B and Pu F C 1996 *J. Phys.: Condens. Matter* **8** L541
[13] Leggett A J, Chakravarty S, Dorsey A T, Fisher M P A, Garg A and Zwerger W 1987 *Rev. Mod. Phys.* **59** 1
[14] Coleman S 1985 *Aspects of Symmetry* (New York: Cambridge University Press) ch 7
[15] Garg A and Kim G-H 1992 *Phys. Rev.* B **45** 12 921
The authors have calculated the tunnelling rate of the model in the presence of an external field, which has a different emphasis to ours.
[16] Fradkin E 1991 *Field Theories in Condensed Matter* (New York: Addison-Wesley) ch 5
[17] Haldane F D M 1983 *Phys. Lett.* **93A** 464
[18] Auerbach A 1994 *Interacting Electrons and Quantum Magnetism* (Berlin: Springer) ch 10
[19] Generally the effective Hamiltonian H_{eff} itself does not have a time-reversal symmetry. Here we introduce a 'time-reversal' (time-reversal-like) operator τ to help us to discuss the degeneracy of the eigenstates, which proves to be very useful.